Kinematics Formulas:
A Mathematical Analysis of Motion

1. The velocity of an object with constant acceleration

Equation: $V_f = V_i + at$

where V_f = final velocity
V_i = initial velocity
a = acceleration
t = time

Ex. 1. If a bus with an acceleration of +2.5 m/s^2 has a velocity of +4.0 m/s at t = 0 s, What is it's velocity after 4.0 s ?

$V_f = ?$
$V_i = 4.0\ m/s$
$a = 2.5\ m/s^2$
$t = 4.0\ s$

$V_f = V_i + at$
$= 4.0\ m/s + (2.5\ m/s^2)(4.0\ s)$
$= 4.0\ m/s + 10.0\ m/s$
$= 14\ m/s$

The final velocity is 14 m/s

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2. The displacement of an object with constant acceleration

Equation: $d = \left(\frac{V_f + V_i}{2}\right)t$ or $d = \frac{1}{2}(V_f + V_i)t$

Ex. 2. What is the displacement of a truck as it is accelerated uniformly from +25.5 km/h to +36.5 km/h in a 15.0 s interval?

To convert from $\frac{km}{h} \rightarrow \frac{m}{s}$ you $\div 3.6$

$d = ?$

$V_i = 25.5\ km/h = 7.08\ m/s$

$V_f = 36.5\ km/h = 10.1\ m/s$

$t = 15.0\ s$

$d = \left(\frac{V_f + V_i}{2}\right)t$

$= \left(\frac{10.1\ m/s + 7.08\ m/s}{2}\right) 15.0\ s$

$= \left(\frac{17.18\ m/s}{2}\right) 15.0\ s$

$= 8.59\ m/s \times 15.0\ s$

$= 129\ m$

3. The displacement when acceleration and time are known

Equation: $d = V_i t + \frac{1}{2} a t^2$

Ex. 3. A skateboarder starting from rest accelerates uniformly at +3.0 m/s^2 for 10.0 s. How far does the skateboarder travel?

$d = ?$
$V_i = 0.0 \, m/s$
$t = 10.0 \, s$
$a = 3.0 \, m/s^2$

$d = V_i t + \frac{1}{2} a t^2$
$= 0.0 \times 10 s + \frac{1}{2}(3.0 \, m/s^2)(10.0 s)^2$
$= 0 + \frac{1}{2}(3.0 \, m/s^2)(100 \, s^2)$
$= 150 \, m$

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4. The velocity when displacement and acceleration are known

Equation: $V_f^2 = V_i^2 + 2ad$

Ex. 4. If a subway train travelling at 65 m/s slows to a stop with an acceleration of -2.2 m/s^2, at what distance from the station does it need to begin decelerating?

BEDMAS

$V_f = 0\,m/s$
$V_i = 65\,m/s$
$a = -2.2\,m/s^2$
$d = ?$

$$V_f^2 = V_i^2 + 2ad$$
$$V_f^2 - V_i^2 = 2ad$$
$$\frac{V_f^2 - V_i^2}{2a} = d$$
$$\frac{(0)^2 - (65\,m/s)^2}{2(-2.2\,m/s^2)} = d$$
$$\frac{0 + 4225\,m^2/s^2}{+4.4\,m/s^2} = d$$
$$9.6 \times 10^2\,m = 960\,m = d$$

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*rearranging

$$V_f = V_i + at \quad \text{for "}t\text{"}$$

BEDMAS

$$\frac{V_f - V_i}{a} = \frac{at}{a}$$

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1. $V_i = 6.0\ m/s$

$V_f = 38.0\ m/s$

$t = 4.0\ s$

$a =$

$\frac{m}{s} \div s$

$\frac{m}{s} \times \frac{1}{s} = \frac{m}{s^2}$

$V_f = V_i + at$

BEDMAS

$\frac{V_f - V_i}{t} = a$

$\frac{38\ m/s - 6.0\ m/s}{4.0\ s} = a$

$\frac{32\ m/s}{4.0\ s} = a$

$8.0\ m/s^2 = a$

3. $t = 3.0\,s$

$a = -8.0\,m/s^2$

$V_i = ?$

$V_f = 0.0\,m/s$

$V_f = V_i + at$ BEDMAS

$V_f - at = V_i$

$0 - (-8.0\,m/s^2)(3.0\,s) = V_i$

$0 - (-24\,m/s) = V_i$

$24\,m/s = V_i$

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4. a) $V_f = 4.0\ m/s$
$t = 2.5\ s$
$d =$
$V_i = 0\ m/s$

$$d = \left(\frac{V_f + V_i}{2}\right)t$$
$$= \left(\frac{4.0\ m/s + 0\ m/s}{2}\right)2.5\ s$$
$$= \left(\frac{4.0\ m/s}{2}\right)2.5\ s$$
$$= 5.0\ m$$

b) $a = ?$ Hint: DON'T USE YOUR ANSWER IF YOU DON'T NEED TO!

$$V_f = V_i + at$$
$$a = \frac{V_f - V_i}{t}$$
$$= \frac{4.0\ m/s - 0\ m/s}{2.5\ s}$$
$$= 1.6\ m/s^2$$

7. $V_i = 20.0\ m/s$

$d = 1.50 \times 10^2\ m = 150\ m$

$t = 10.0\ s$

$V_f =$

BEDMAS ←

$$d = \left(\frac{V_f + V_i}{2}\right)t$$

$$\frac{d}{t} = \frac{(V_f + V_i)}{2}$$

$$\frac{2d}{t} = V_f + V_i$$

$$\frac{2d}{t} - V_i = V_f$$

$$\frac{2(150\ m)}{10.0\ s} - 20.0\ m/s = V_f$$

$$30.0\ m/s - 20.0\ m/s = V_f$$

$$10.0\ m/s = V_f$$

14. $V_i = 5.6\ m/s$

$a = 0.60\ m/s^2$

$t = 4.0\ s$

a) $d = ?$

$$d = V_i t + \frac{1}{2} a t^2$$

$$= 5.6\ m/s \times 4.0\ s + \frac{1}{2}(0.60\ m/s^2)(4.0\ s)^2$$

$$= 22.4\ m + 4.8\ m$$

$$= 27\ m$$

b) $V_f = ?$

$$V_f = V_i + at$$

$$= 5.6\ m/s + (0.60\ m/s^2)(4.0\ s)$$

$$= 8.0\ m/s$$

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15. $V_f = 14\ m/s$

$V_i = 22\ m/s$

$d = 125\ m$

a) $a = ?$

$V_f^2 = V_i^2 + 2ad$ BEDMAS ←

$V_f^2 - V_i^2 = 2ad$

$\frac{V_f^2 - V_i^2}{2d} = a$

$\frac{(14\ m/s)^2 - (22\ m/s)^2}{2(125\ m)} = a$

$\frac{196\ m^2/s^2 - 484\ m^2/s^2}{250\ m} = a$

$\frac{-288\ m^2/s^2}{250\ m} = -1.2\ m/s^2 = a$

b) $t = ?$

$d = \left(\frac{V_f + V_i}{2}\right)t$

$\frac{2d}{V_f + V_i} = \frac{\left(V_f + V_i\right)}{\left(V_f + V_i\right)}t$

$\frac{d}{\left(\frac{V_f + V_i}{2}\right)} = t$

$\frac{125\ m}{\left(\frac{14\ m/s + 22\ m/s}{2}\right)} = t$

$\frac{125\ m}{18\ m/s} = t$

$6.9\ s = t$